DETERMINATION OF CHARACTERISTIC DYNAMIC AND THERMAL INTERACTION TIMES IN PROBLEMS OF GAS SUSPENSION WAVE DYNAMICS

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In the solution of wave dynamics problems of gas suspensions it is useful to employ the concepts of the characteristic time τ and length l of interphase exchange of momentum (τ_v, l_v) and heat (τ_T, l_T), these being the characteristic times and lengths at which the differences in velocities and temperatures of the gas and particles change by a specific number of times. Comparison of these values to the characteristic time and linear scale of the problem permits conclusions as to the possibility of flow description within the framework of simple special models (the gas suspension thermodynamic equilibrium model, single-temperature gas suspension model, etc.). If the flow must be studied within the framework of the generalized two-temperature, two-velocity model, then the characteristic interphase interaction times and lengths permit introduction of convenient dimensionless variables. It will be shown below that the characteristic times for dynamic τ_v and thermal τ_T phase interaction are determined by the characteristic Reynolds numbers for flow over particles. They may differ greatly from each other, and from the conventional Stokes times [1, 2] corresponding to small Reynolds numbers.

Let an isolated spherical particle of diameter d and density ρ_s having an initial velocity v_{s0} and temperature T_{s0} be located within a gas flow behind the front of a shock wave moving with velocity D, the gas flow velocity relative to a fixed coordinate system being v_{gf}, with temperature T_{gf} and density ρ_{gf} (the situation shown schematically in Fig. 1). In such a constant flow the laws specifying the changes in particle velocity and temperature are described by differential equations

$$m dv_s/dt = f,\ mc_s dT_s/dt = q\ (m = (1/6)\pi d^3\rho_s),$$
$$f = 0.125\pi d^2\rho_{gf}C_d|v_{gf} - v_s|(v_{gf} - v_s),\ q = \pi d\lambda_g \mathrm{Nu}(T_{gf} - T_s), \quad (1)$$

where C_d is the resistance coefficient, Nu is the heat exchange parameter, ρ, λ, c, the density, thermal conductivity, and specific heat, the subscripts g and f denote gas and particle parameters.

The resistance coefficient C_d and heat exchange parameter Nu, whcih determine the intensity of thermal and mechanical interaction of the particle with the gas behind the shock wave front, depend on the characteristic Reynolds and Mach numbers of the relative motion [3]. However, particle flowover regimes with large Mach numbers are realized only in very intense shock waves, while in other cases the Mach effect is small and may be neglected. We take

$$C_d = 24/\mathrm{Re} + 4/\mathrm{Re}^{0.5} + 0.44,\ \mathrm{Nu} = 2 + 0.6\mathrm{Re}^{0.5}\mathrm{Pr}^{0.3},\quad \mathrm{Re} = |v_g - v_s|d\rho_g/\mu_g,\quad \mathrm{Pr} = \mu_g c_{pg}/\lambda_g, \quad (2)$$

where c_{pg} and μ_g are the specific heat at constant pressure and the dynamic viscosity; Pr is the Prandtl number of the gas.

At low Re (Re < ~1) a Stokes flowover regime is realized, in which in place of Eq. (2) we have $C_d \cong 24/\mathrm{Re}$, $\mathrm{Nu} \cong 2$. In this case in accordance with Eq. (1) the change in velocity v_s and temperature T_s of the particle with time are defined by exponential functions

$$\frac{\Delta v}{\Delta v_0} = \frac{v_s - v_{gf}}{v_{s0} - v_{gf}} = \exp\left(-\frac{t}{\tau_v^S}\right),\ \frac{\Delta T}{\Delta T_0} = \frac{T_s - T_{gf}}{T_{s0} - T_{gf}} = \exp\left(-\frac{t}{\tau_T^S}\right)$$
$$\left(\tau_v^S = \frac{\rho_s d^2}{18\mu_g},\quad \tau_T^S = \frac{\rho_s c_s d^2}{12\lambda_g}\right), \quad (3)$$

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where τ_v^S, τ_T^S are the characteristic times over which the difference between gas and particle velocities and temperatures change by a factor of $e \cong 2.7$ times in the Stokes regime of relative motion. These parameters have meaning over the entire zone of parameter equalization and therefore can be called relaxation times (τ_v^S is the Stokes velocity relaxation time, τ_T^S is the Stokes temperature relaxation time). We note that

$$\frac{\tau_T^S}{\tau_v^S} = k, \quad k = 1.5\frac{c_s}{c_{pg}}\,\mathrm{Pr} \sim 1. \tag{4}$$

The constant k is of the order of magnitude of unity, since $c_s \sim c_{pg}$, while the gas Prandtl number $\mathrm{Pr} \sim 1$.

If the Reynolds numbers of the relative flow over the particles are sufficiently great ($\mathrm{Re} > 10^2$) we have Newtonian flow over the particles, wherein with consideration that $\mathrm{Pr} \sim 1$ in accordance with Eq. (2) we may take

$$C_d \cong 0.44, \ \mathrm{Nu} \cong 0.6\mathrm{Re}^{1/2}. \tag{5}$$

Integration of equation of motion (1) with resistance law (5) gives a time dependence for particle velocity

$$\frac{\Delta v}{\Delta v_0} = \frac{v_s - v_{gf}}{v_{s0} - v_{gf}} = \left[1 + (e-1)\frac{t}{\tau_v^N}\right]^{-1}, \quad \tau_v^N \cong 5.2\frac{\rho_s}{\rho_{gf}}\frac{d}{|v_{gf} - v_{s0}|}, \tag{6}$$

where τ_v^N is the characteristic time over which the difference between gas and particle velocities in the initial section of particle motion behind the shock wave changes by a factor of e times with Newtonian overflow.

In contrast to Eq. (3), Eq. (6) is not exponential, and τ_v^N is not a true relaxation time having meaning over the entire zone of velocity equalization. We note that in contrast to the Stokes time τ_v^S the Newtonian time τ_v^N is proportional not to the square, but to the first power of the particle diameter d. Moreover, it depends on the characteristic phase density ratio ρ_s/ρ_{gf} and the characteristic velocity difference $|v_{gf} - v_{s0}|$. Then

$$\frac{\tau_v^N}{\tau_v^S} \cong \frac{94}{\mathrm{Re}_*}, \quad \mathrm{Re}_* = \frac{|v_{gf} - v_{s0}|\, d\rho_{gf}}{\mu_g} > 10^2, \tag{7}$$

where Re_* is the characteristic Reynolds number of the relative motion, defined by the gas and particle parameters behind the shock wave front. Thus the characteristic Newtonian time τ_v^N is always less than the Stokes time τ_v^S.

In the case of Newtonian flow over the particles we obtain the particle temperature dependence on time by integration of the heat increment equation (1) with heat exchange law (5) with consideration of changes in particle velocity with time (number Re) in accordance with Eq. (6). We have

$$\frac{\Delta T}{\Delta T_0} = \frac{T_s - T_{gf}}{T_{s0} - T_{gf}} \cong \exp\left\{0.35\sqrt{\mathrm{Re}_*}\,\frac{\tau_v^N}{\tau_T^S}\left[1 - \left(1 + 1.72\frac{t}{\tau_v^N}\right)^{1/2}\right]\right\}. \tag{8}$$

Equation (8) permits derivation of an expression for the characteristic temperature equalization time for Newtonian flow over the moving particle (the time over which the difference between gas and particle temperatures changes by a factor of e = 2.7 times):

$$\frac{\tau_T^N}{\tau_v^N} \cong 3.4\cdot 10^{-2} k\sqrt{\mathrm{Re}_*} + 5.2\cdot 10^{-4} k^2 \mathrm{Re}_* \left(k = 1.5\frac{c_s}{c_{pg}}\,\mathrm{Pr} \sim 1, \ \mathrm{Re}_* > 10^2\right). \tag{9}$$

It is evident that in contrast to the Stokes case, where in accordance with Eq. (4) $\tau_T^S/\tau_v^S \sim 1$, the values of the ratios τ_T^N and τ_v^N depend on the characteristic Reynolds number Re_* for Newtonian flow over the particles, and at Re_* of the order of 10^3 and above may significantly exceed unity. We note that τ_T^N, just like τ_v^N, characterizes only the Newtonian zone of temperature equalization and (in contrast to τ_T^S) is not the true temperature relaxation time. In contrast to the Stokes time τ_T^S depends on the characteristic Reynolds number of the relative motion Re_*. We then have

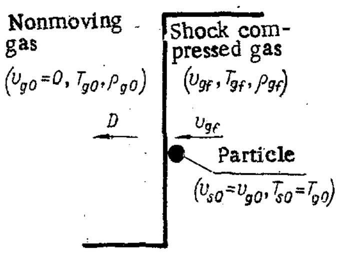

Fig. 1

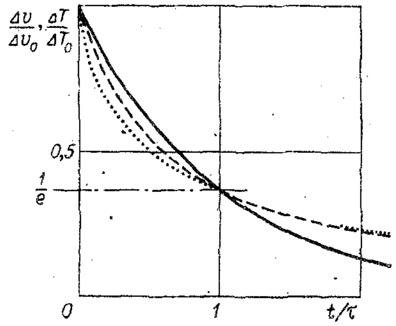

Fig. 2

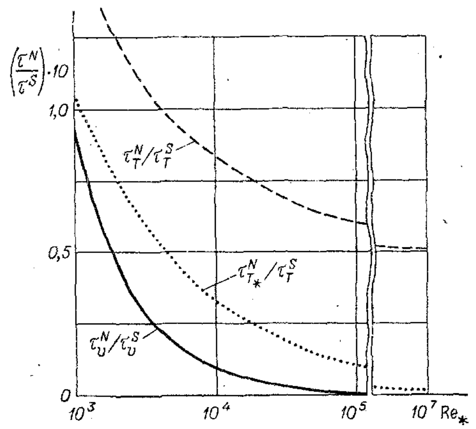

Fig. 3

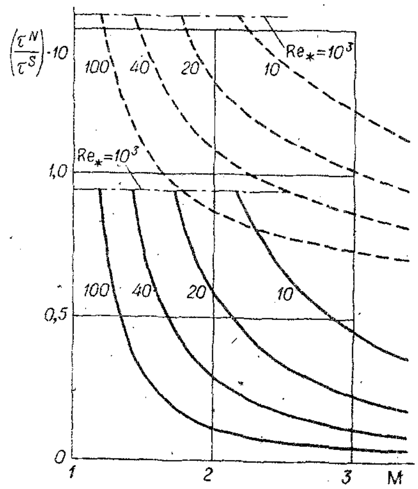

Fig. 4

$$\frac{\tau_T^N}{\tau_T^S} \cong 0.05k + \frac{3.3}{\sqrt{\mathrm{Re}_*}} \left(k = 1.5 \frac{c_s}{c_{pg}} \mathrm{Pr} \sim 1,\ \mathrm{Re}_* \gtrsim 10^2\right),$$

consequently, the characteristic Newtonian time τ_T^N is significantly smaller than the corresponding Stokes time τ_T^S. However, the value of the ratio τ_T^N/τ_T^S cannot be less than 0.05 no matter how high Re_*, i.e., the characteristic Newtonian temperature change time τ_T^N cannot be arbitrarily small (in contrast to the Newtonian velocity change time, which tends to zero upon increase in Re_* [see Eq. (7)].

We note that if an expression for τ_T^N is obtained without consideration of the effect of particles being set in motion under the action of the gas flow, setting the heat exchange parameter Nu in Eq. (5) equal to $0.6\mathrm{Re}_*^{1/2}$ = const, then at values $\mathrm{Re}_* \gg 10^3$ large errors may be produced in determining τ_T^N. In fact, with such an approach to determining τ_T^N (we will denote the corresponding value of τ_T^N by the subscript * below) we have

$$\tau_{T*}^N/\tau_T^S \cong 3.3/\sqrt{\mathrm{Re}_*}.$$

In contrast to τ_T^N, the quantity $\tau_{T*}^N \to 0$ as $\mathrm{Re}_* \to \infty$. Therefore, for example, at $\mathrm{Re}_* = 10^5$ we have $\tau_T^N \doteq 6\tau_{T*}^N$, and with increase in Re_* the difference between τ_T^N and τ_{T*}^N also increases.

With consideration of Eqs. (4), (7), and (9), which relate the quantities τ_T^N, τ_v^N to each other, the law of particle temperature change with time, Eq. (8), can be rewritten in the form

$$\frac{\Delta T}{\Delta T_0} = \frac{T_s - T_{gf}}{T_{s0} - T_{gf}} \cong \exp\left\{\frac{1}{0.03k\sqrt{\mathrm{Re}_*}}\left[1 - \sqrt{1 + \left[(1 + 0.03k\sqrt{\mathrm{Re}_*})^2 - 1\right]\frac{t}{\tau_T^N}}\right]\right\}. \quad (10)$$

It can easily be shown that for $\mathrm{Re}_* \gtrsim 10^5$ (when $0.03k\sqrt{\mathrm{Re}_*} \gg 1$) for times $t/\tau_T^N \gtrsim 1$, the parameter Re_* has practically no effect on the form of Eq. (10), and

$$\Delta T/\Delta T_0 \cong \exp\left\{-\sqrt{t/\tau_T^N}\right\}.$$

The form of the dimensionless time dependences of relative particle velocity and temperature behind the wave front for Stokes and Newtonian overflow regimes is illustrated in Fig. 2, where the solid line is the Stokes exponential dependence, Eq. (3), the dashed line is Newtonian velocity dependence (6), and the dotted line is particle temperature vs time, Eq. (10), at $Re_* = 10^5$. Each curve is constructed using its own dimensionless time value. It is evident that Eqs. (6), (10) do not differ greatly from exponential, although they are steeper for $t/\tau < 1$ and flatter for $t/\tau > 1$. At $t/\tau > 1$, Eqs. (6), (10) (dashed and dotted lines) practically coincide (as was noted above, at $Re_* > 10^5$ the form of Eq. (10) does not depend on this parameter). Figure 3 illustrates the dependence of characteristic Newtonian particle velocity and temperature change times on characteristic Reynolds number of the relative flowover Re_* and clearly shows the difference between these times over a wide Re_* range.

Figure 4 shows the dependence of characteristic Newtonian particle velocity change time (relative to the Stokes velocity relaxation time, solid lines) and the characteristic temperature change time (relative to the Stokes temperature relaxation time, dashed lines) on Mach number for the case of a shock wave in air. The various curves correspond to different particle diameters (numbers along the curves are diameters in 1). Each curve illustrates the corresponding functions in the region of Mach numbers sufficiently large for a given diameter, at which there is no doubt that a Newtonian regime is realized behind the front ($Re_* > 10^3$). With increase in wave intensity the ratio between Newtonian and Stokes times decreases, with the characteristic velocity change time tending to zero, and the temperature change time to approximately 0.05. For waves of fixed intensity the ratio of Newtonian to Stokes times increases with decrease in particle diameter, while

$$\tau_v^N/\tau_v^S \sim Ad^{-1}, \quad \tau_T^N/\tau_T^S \sim 0.05 + Bd^{-1/2}\,(A, B = \text{const}).$$

The results of the analysis performed may prove useful in determining characteristic momentum and heat exchange times between phases in problems of dynamics of gas suspensions.

LITERATURE CITED

1. F. Marble, "Dynamics of dusty gases," Ann. Rev. Fluid Mech., No. 2 (1970).
2. L. G. Loitsyanskii, Mechanics of Liquids and Gases [in Russian], Nauka, Moscow (1973).
3. A. I. Ivandaev, A. G. Kutushev, and R. I. Nigmatulin, "Gas dynamics of multiphase media," VINITI Ser. Mekh. Zhidk. Gaza, 16 (1981).